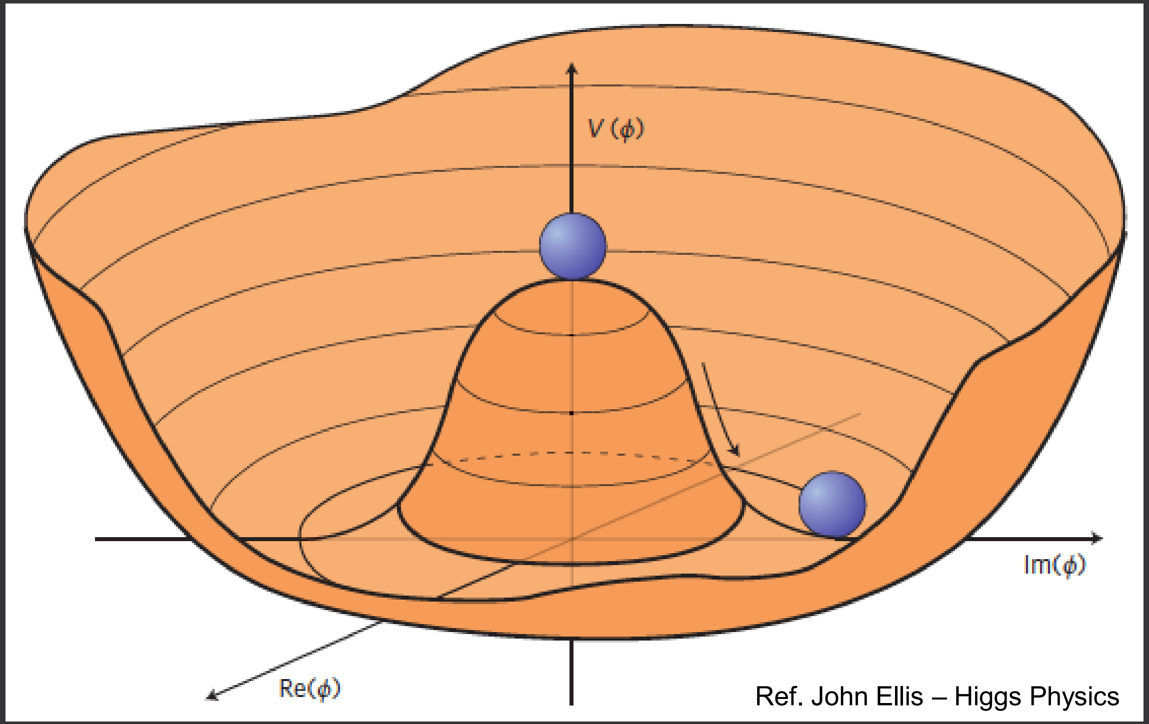
The Higgs as the Inflaton

Patrick Adams, Advisor: Daniel Grin

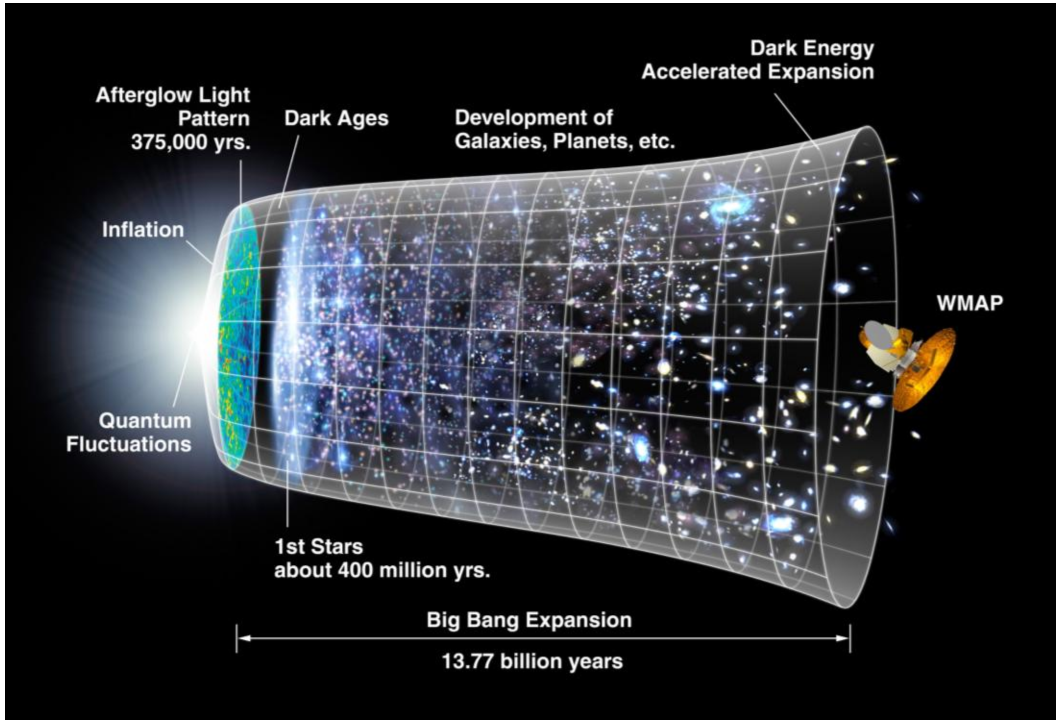


Our Collaborators

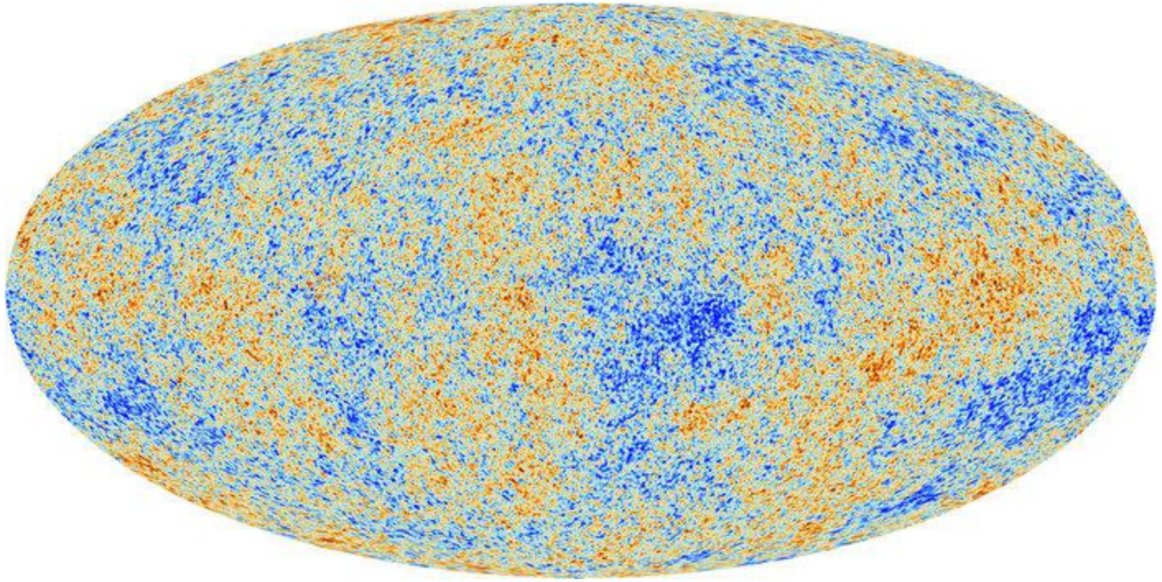
- **David Kaiser** – MIT – Wrote detailed notes on Higgs inflation
- **Renée Hložek** – University of Toronto – Project used Fisher analysis to test how compatible a simplified New Higgs inflation is with the CMB
- **William Koster** – Haverford Alum – Thesis work with Prof. Hložek inspired this project
- **Chanda Prescod-Weinstein** – Brown University

Steady State to Big Bang

- Detection of the Cosmic Microwave Background (CMB) suggests there was a Big Bang
- The universe expanded from a singularity
- Expanded from more dense state to less dense state



Ref. NASA/WMAP Science Team

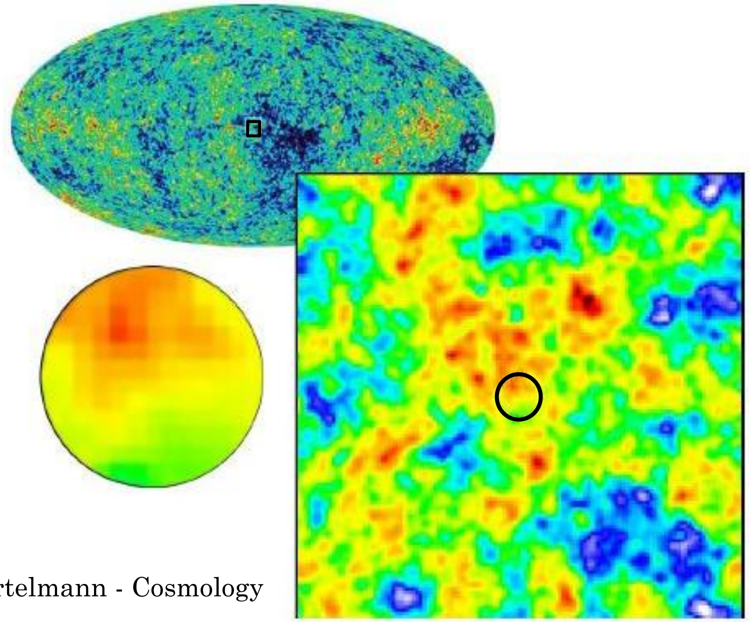


Ref. ESA and the Planck Collaboration

Problems with the Big Bang

Horizon Problem

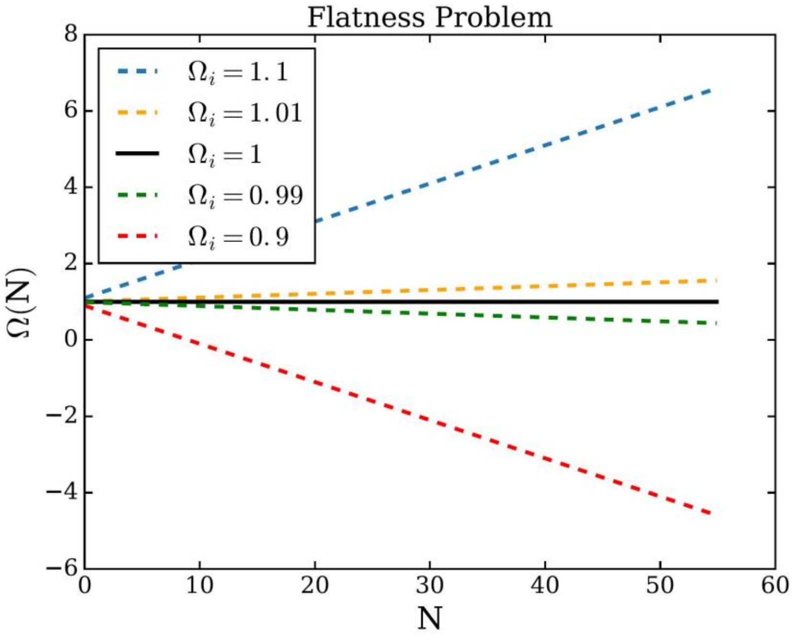
- CMB is isotropic and homogenous: about the same temperature
- The "particle horizon" is only about 1°, but cosmic horizon must have been much larger



Ref. Bartelmann - Cosmology

Flatness Problem

- Flat universe today: More flat in past
- In a matter and radiation dominated universe, perturbation size would only increase



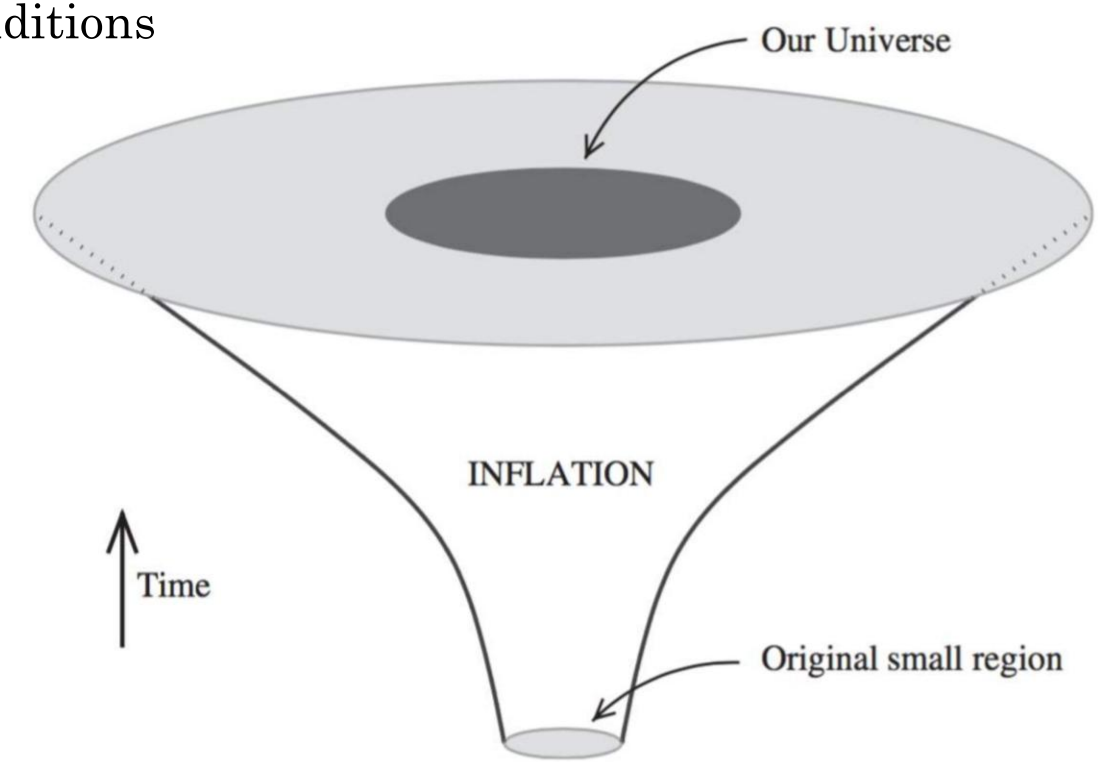
Inflation as a Solution

- Epoch of accelerated expansion of the universe
- Inflation allows for a period in which the universe is in causal contact
- Avoids need for very specific initial conditions
- First Proposed by Prof. Alan Guth

N is E-Folding Value:

$$a(t) = e^{\int_{t_i}^{t} H dt'}$$

$$= e^{N}$$

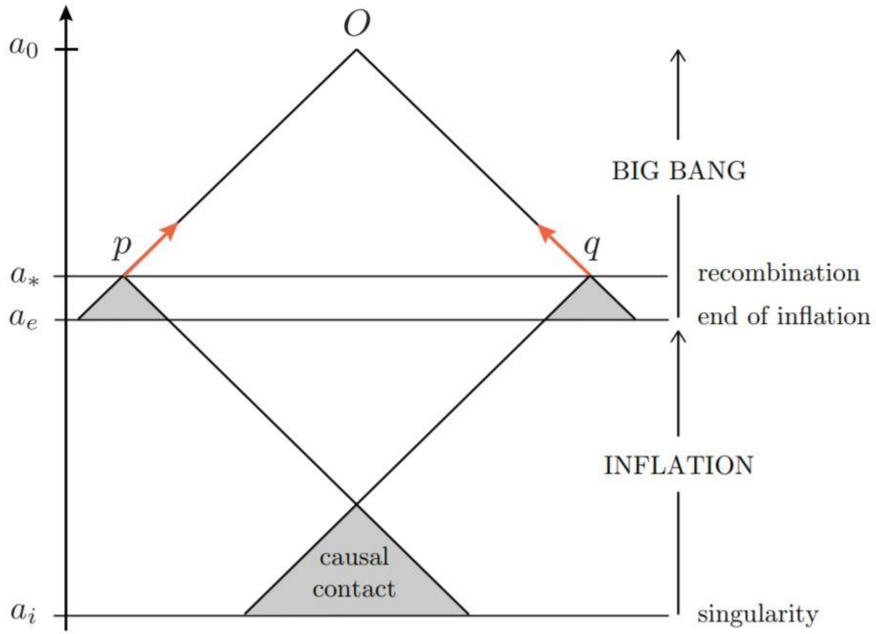


Ref. Liddle – Cosmological Inflation

"Shrinking" Causal Horizon

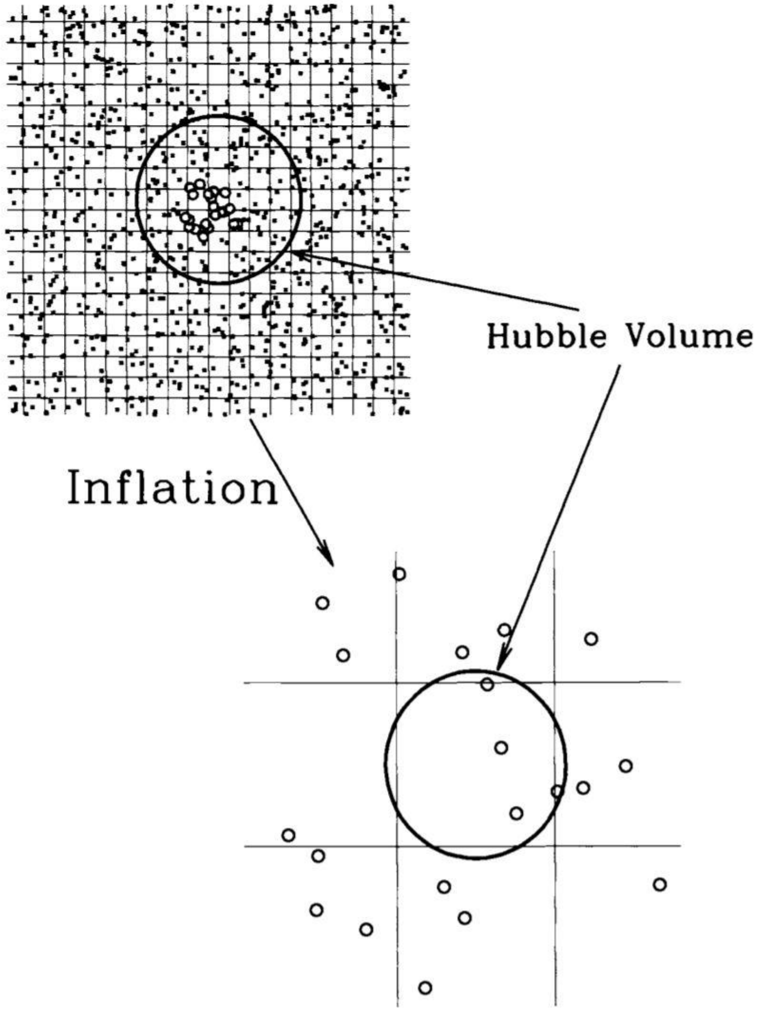
- Universe's co-moving coordinates expanded during inflation – effective causal horizon stays the same

- Thus, the effective causal horizon that matter can interact with shrinks



Ref. Baumann – TASI Lectures on Inflation

Ref. Dodelson – Modern Cosmology

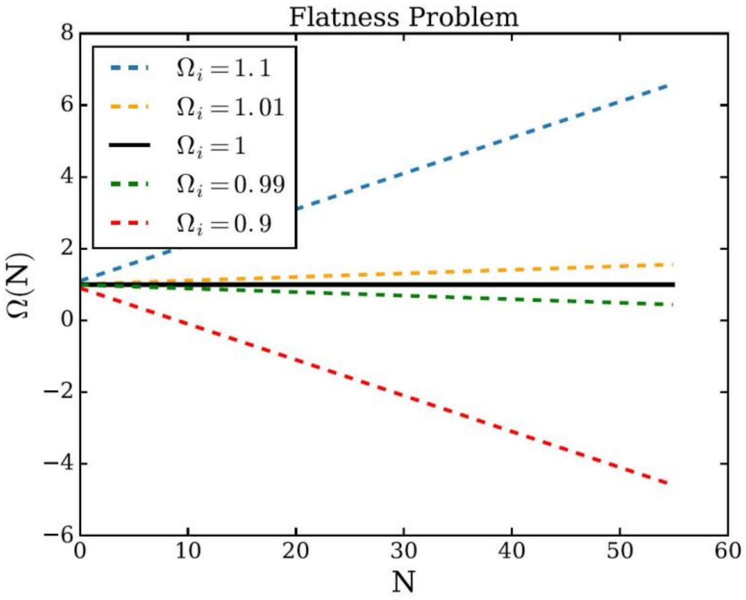


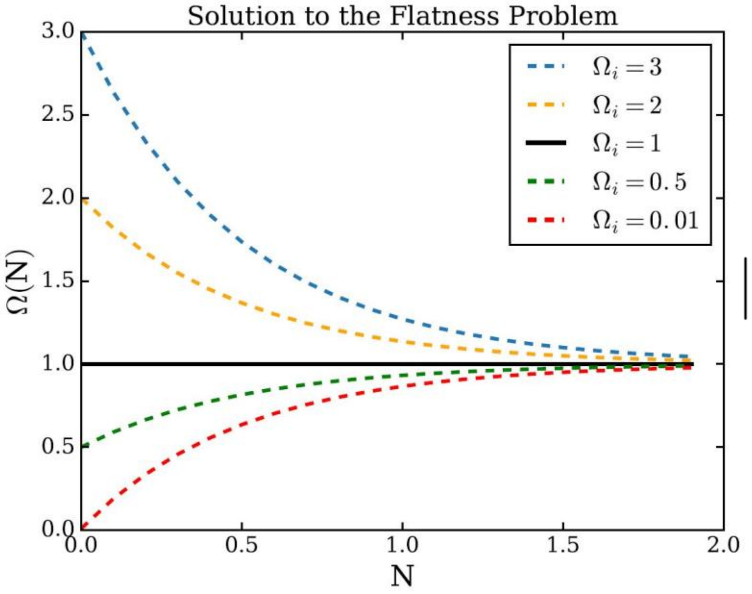
Inflating Universe to Flat Universe

- Density perturbations change exponentially during inflation, thus initial conditions converge to a flat universe
- A radiation or matter dominated universe tends to increase the relative size of perturbations

$$|1-\Omega(t)| = \frac{k}{a(t)^2 H(t)^2}$$

$$|1-\Omega(t)| \propto t$$





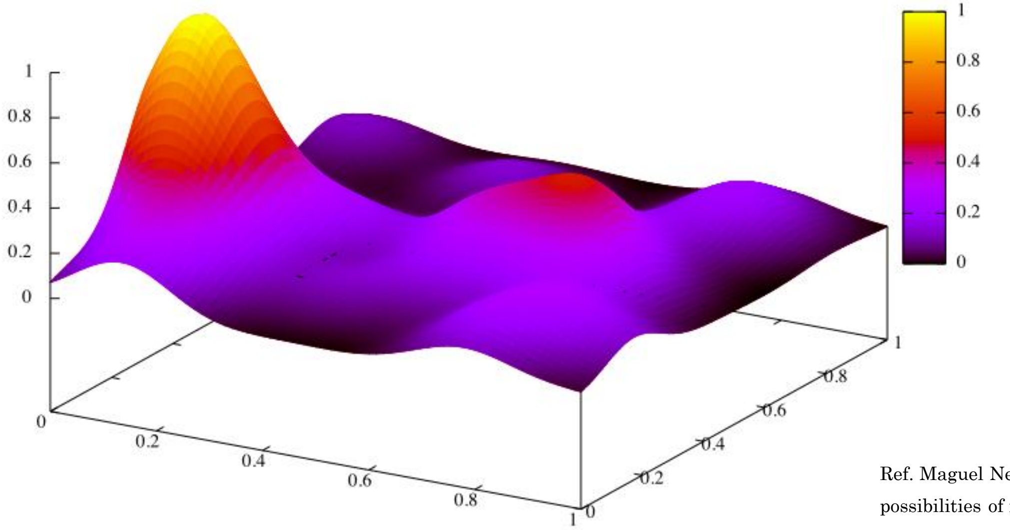
$$|1-\Omega(t)| \propto e^{Ht}$$

Inflation described by a scalar field

- Homogenous field plus a perturbed field:

$$\phi(t,\vec{x}) = \varphi(t) + \delta\phi(t,\vec{x})$$

- Associated spin-0 particle: the ***Inflaton***
- Ex. Gravitational Potential, Temperature

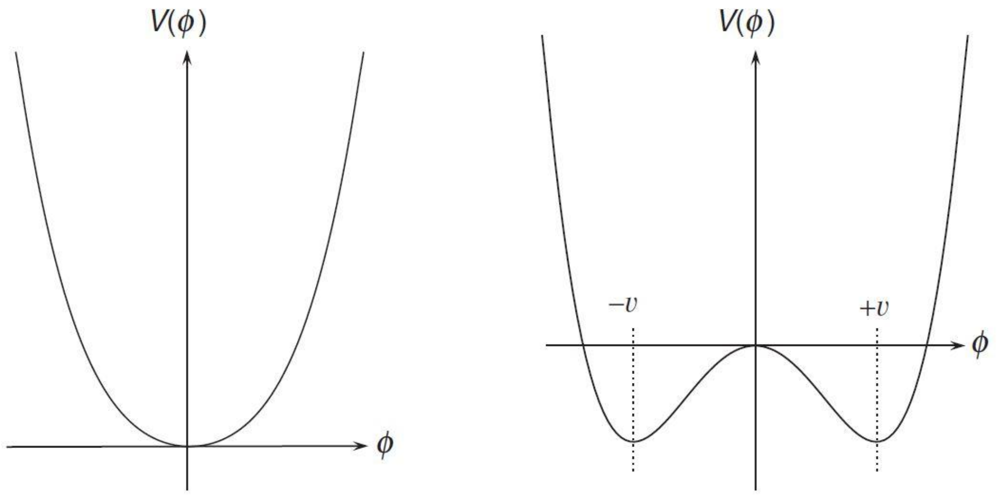


Ref. Maguel Negrão - The challenges and possibilities of real-time Wave Field Synthesis

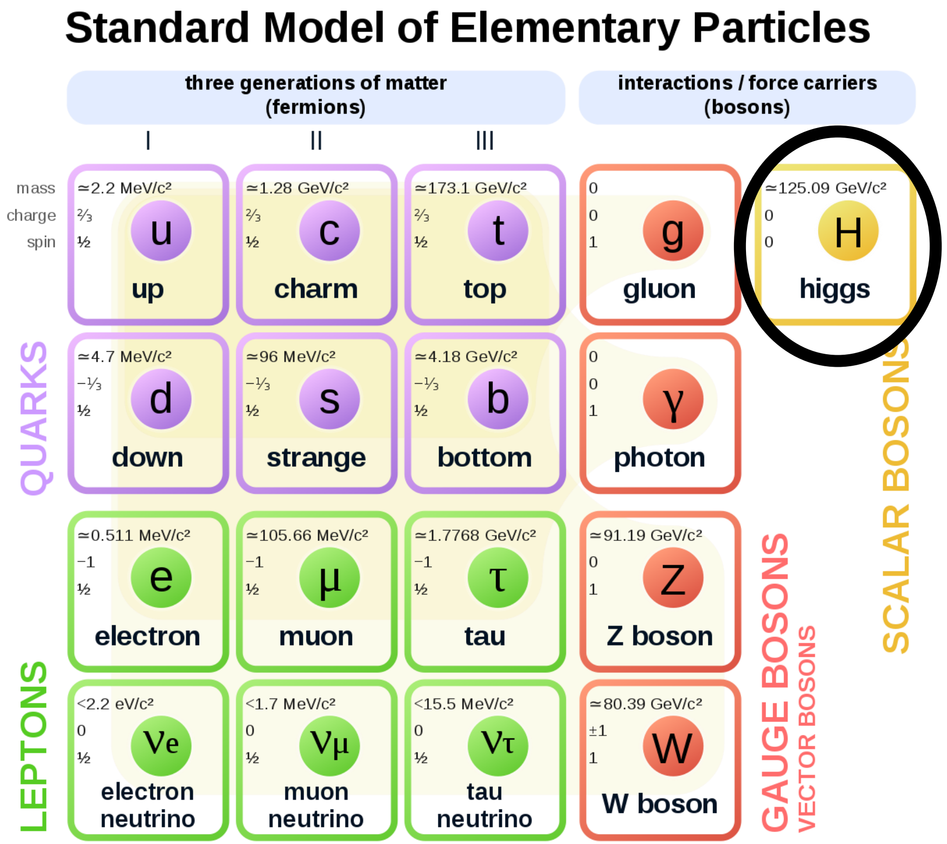
Higgs Boson as the *Inflaton*

- Higgs Boson is only known scalar field and spin-0 particle
- Discovered in 2012 at CERN, questions linger about whether it is a candidate for inflation
- Shape of Higgs Potential is ideal for inflation

$$V(\phi) = \lambda(\phi^2 - M^2)^2$$



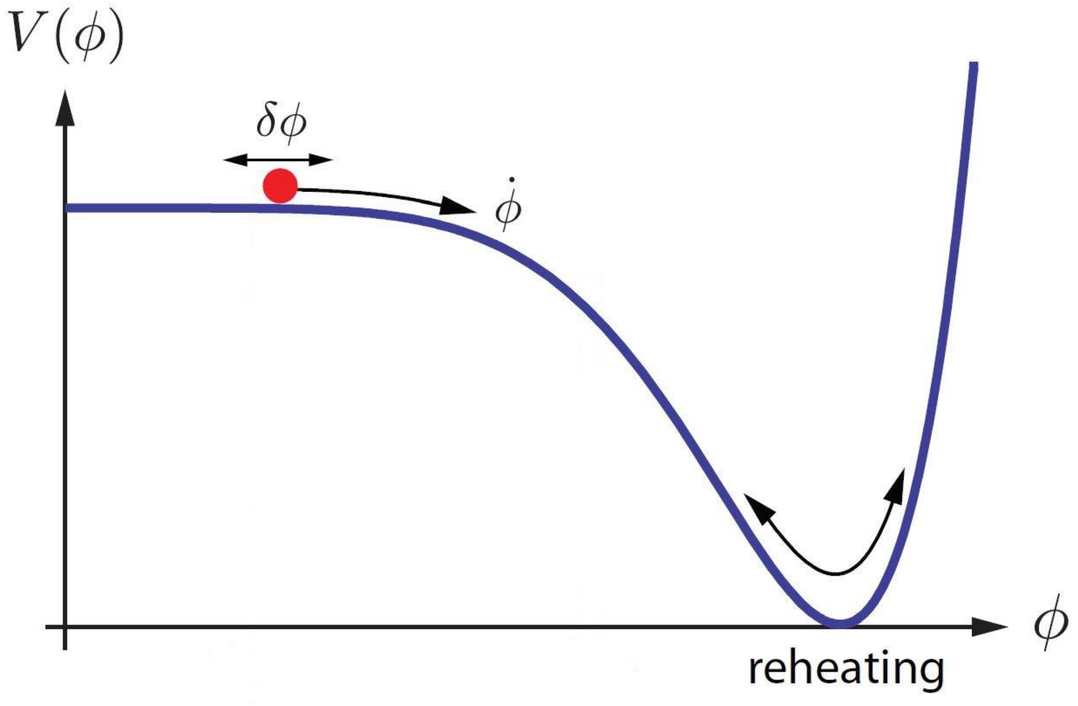
Ref. Thomson – Modern Particle Physics



Ref. Wikimedia Commons

Expansion as a “slow-roll”

- Inflation’s expansion is controlled by the shape of field’s potential
- Slow-roll parameters yield dynamics



Slow-Roll Conditions

$$|\epsilon(\phi)| \ll 1$$

$$|\eta(\phi)| \ll 1$$

Slow-Roll Parameters

$$\epsilon(\phi) = \frac{M_{\text{pl}}^2}{2}\left(\frac{1}{V(\phi)}\frac{dV(\phi)}{d\phi}\right)^2$$

$$\eta(\phi) = M_{\text{pl}}^2\left(\frac{1}{V(\phi)}\frac{d^2V(\phi)}{d\phi^2}\right)$$

Ref. Baumann – TASI Lectures on Inflation

Inflation's end

- Dictated by the Friedman Equation and equation of state parameter w:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^4}(\rho + 3p) \qquad w = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

- Inflation occurs when pressure dominates over matter density

$$3p < -\rho \Rightarrow w < -\frac{1}{3}$$

Equations of Motion

- **"Old Higgs" model** for inflation – assumes GR holds for all time

$$S = \int d^4x\sqrt{-g}\left(\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right)$$

- Extremizing the act yields the homogeneous Equations of Motion

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial\phi} = 0$$

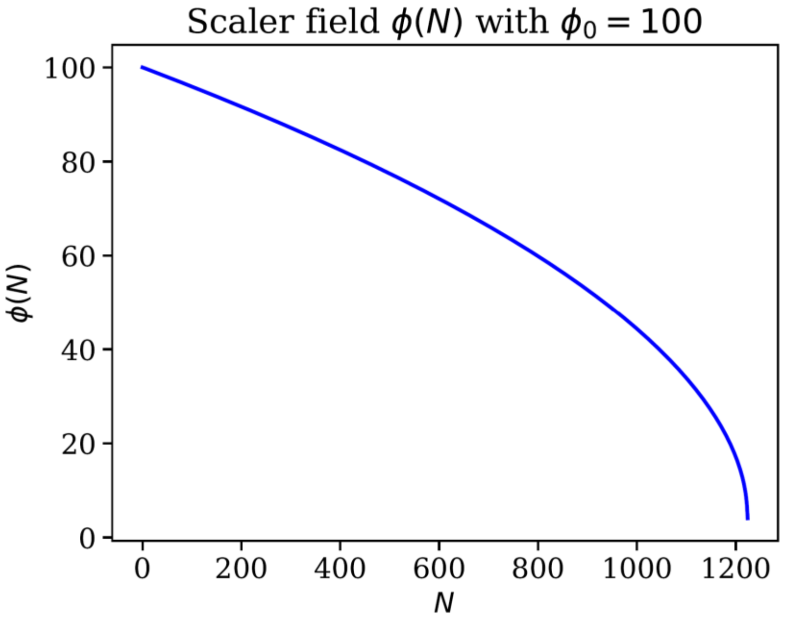
$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

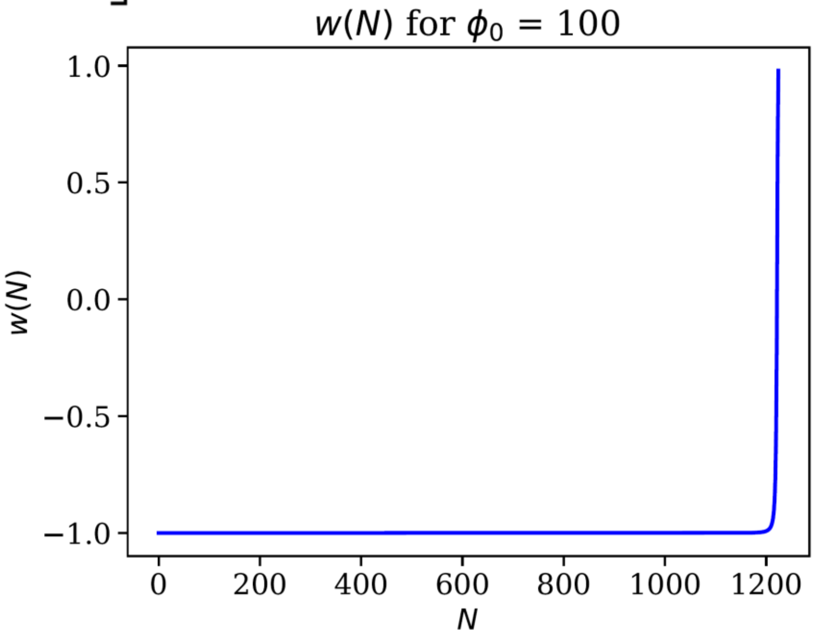
Rewriting Equations to see Inflation

- Rewrote equations in terms of E-folding parameter N for Higgs potential

$$\tilde{H}^2 \frac{d^2\tilde{\varphi}}{dN^2} + (3\tilde{H} + \frac{d\tilde{H}}{dN})\tilde{H}\frac{d\tilde{\varphi}}{dN} + 4\lambda(\tilde{\varphi}^3 - \frac{M^2}{M_{\rm pl}^2}\tilde{\varphi}) = 0$$

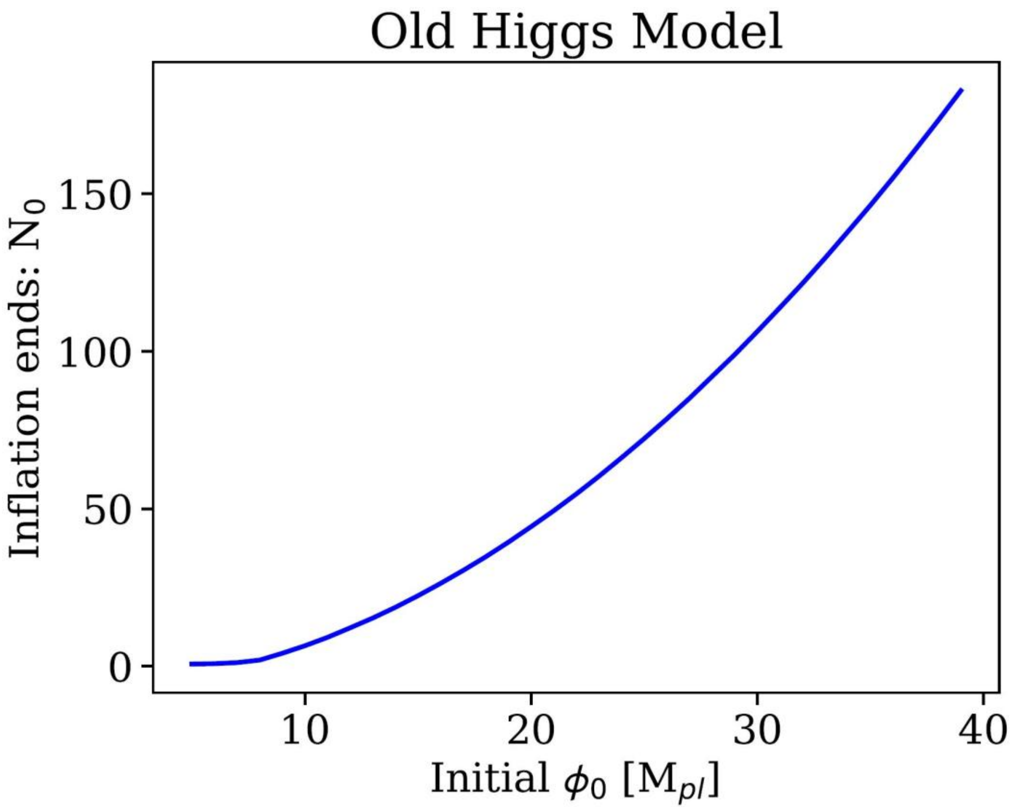
$$\tilde{H} = \left[\frac{\lambda(\tilde{\varphi}^2 - \frac{M^2}{M_{\rm pl}^2})^2}{3 - \frac{1}{2}(\frac{d\tilde{\varphi}}{dN})^2}\right]^{1/2}$$

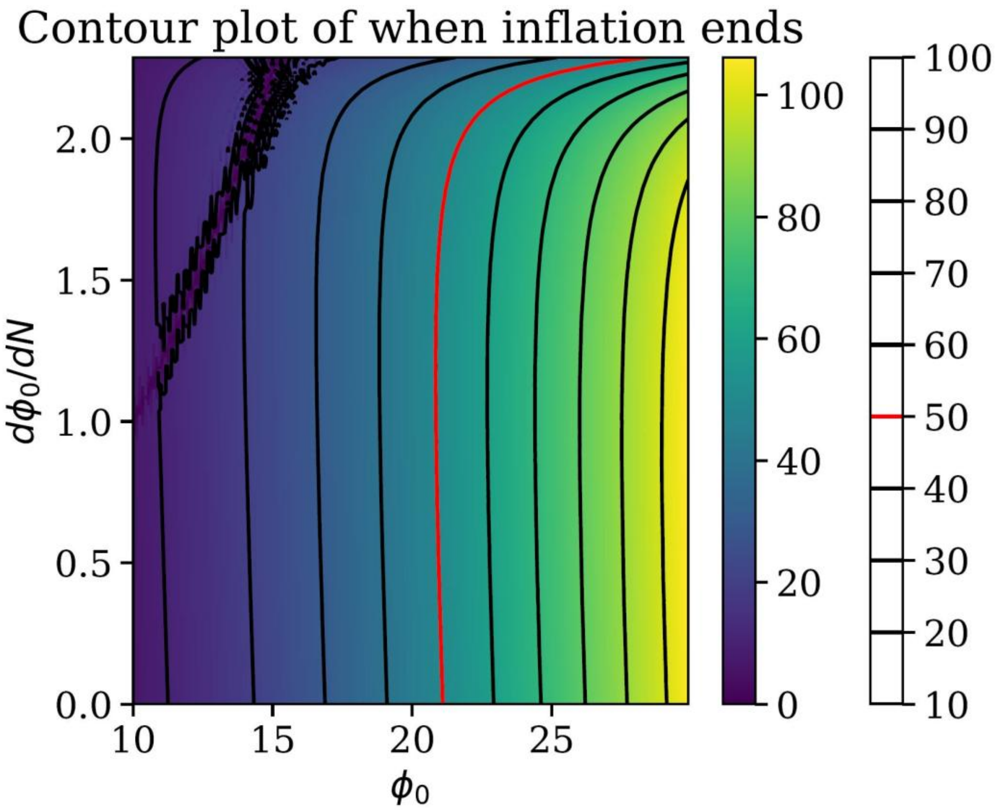




Old Higgs Phase Space

- Solved for inflation's end for different initial scalar field values
- Additionally varied initial $\frac{d\phi}{dN}$ versus $\phi_0$value





Alas, Old Higgs is no good

- Allows for inflating universe but is constrained by particle physics
- Higgs self interaction constant is $0.11 \leq \lambda < 0.27$

$$H^2 \approx \frac{\lambda}{12M_{\mathrm{pl}}^2}\phi^4$$

$$\dot{\phi} \approx -\frac{\lambda}{3H}\phi^3$$

- Applying a quantum gravity and slow-roll constraint yields:

$$\lambda \ll 10^{-2}$$

Non-minimal coupling

- **"New Higgs"** model's couples its kinetic energy term directly to the Einstein curvature tensor

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}\left(g^{\mu\nu} - \frac{1}{\mu^2}G^{\mu\nu}\right)\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

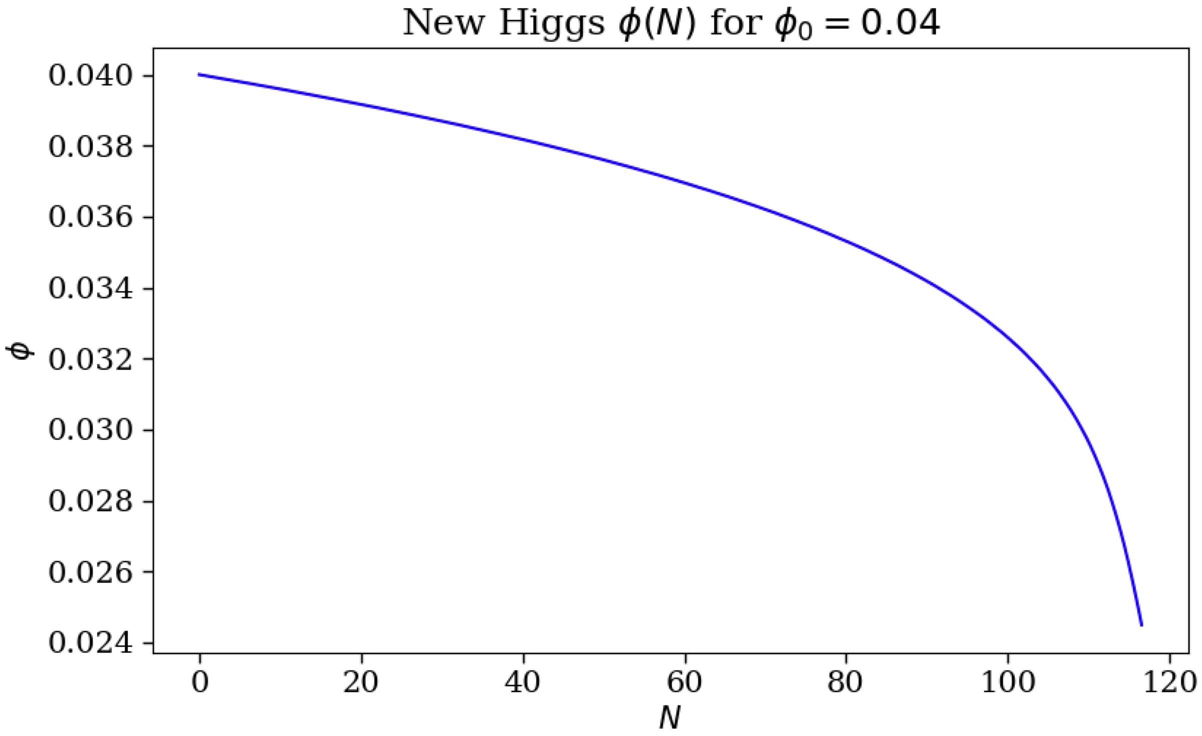
- In the limit of $\phi \gg \frac{1}{\mu^2}$, the EOM looks like the Old Higgs Equations

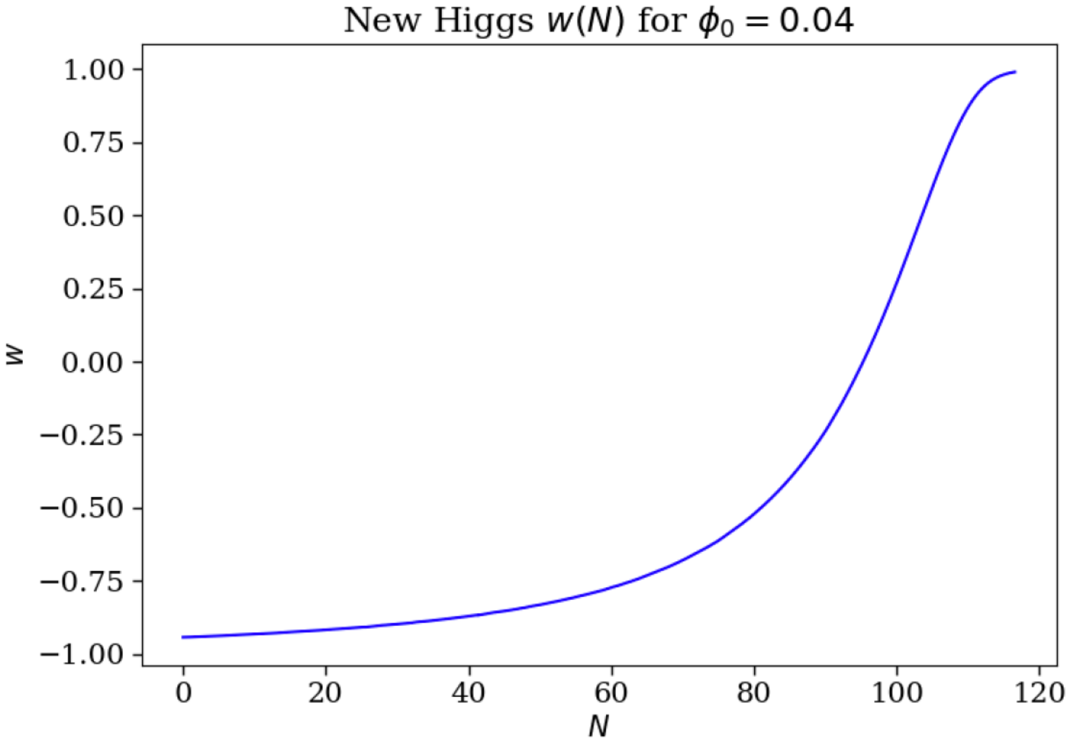
$$\ddot{\varphi}\left[1 + \frac{3H^2}{\mu^2}\right] + 3H\dot{\varphi}\left[1 + \frac{1}{\mu^2}\left(3H^2 + 2\dot{H}\right)\right] + \frac{\partial V(\varphi)}{\partial\varphi} = 0$$

$$H^2 = \frac{1}{3M_{\rm pl}^2}\left[\frac{1}{2}\dot{\varphi}^2\left(1 + \frac{9H^2}{\mu^2}\right) + V(\varphi)\right]$$

New Higgs Dynamics

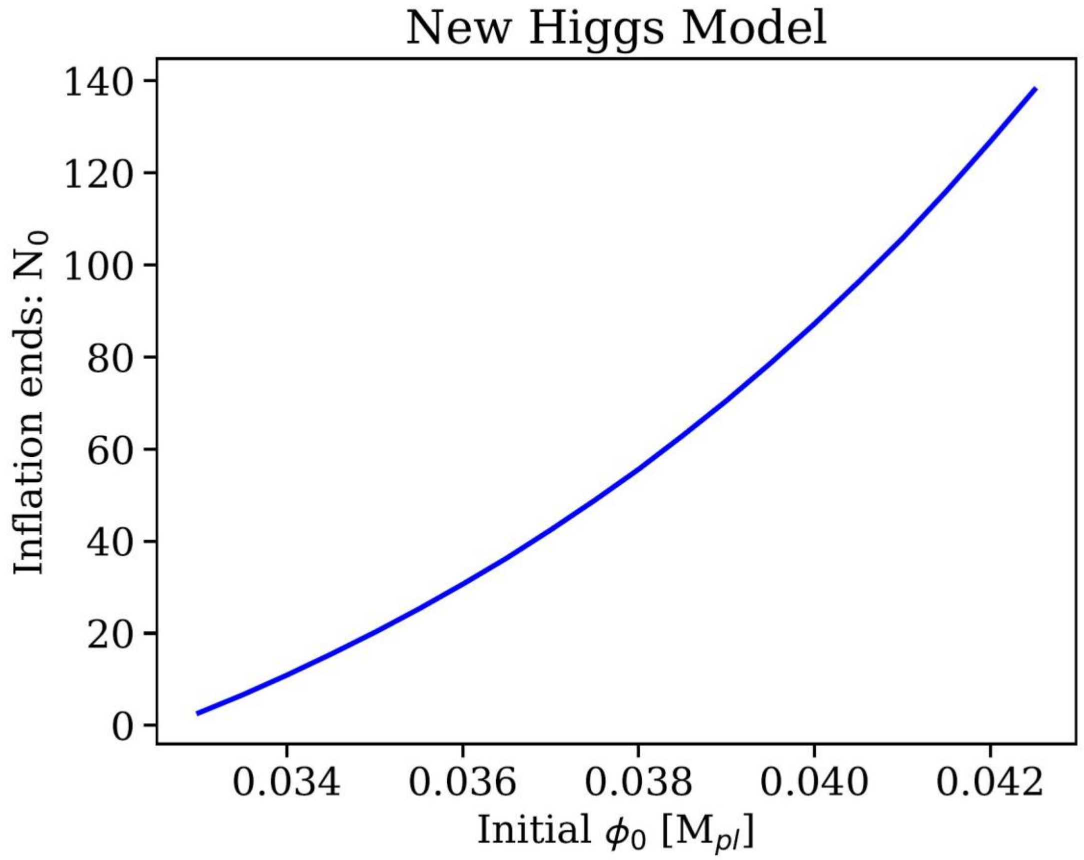
- I solved for the New Higgs equations directly
- This required rewriting the equations in terms of e-foldings

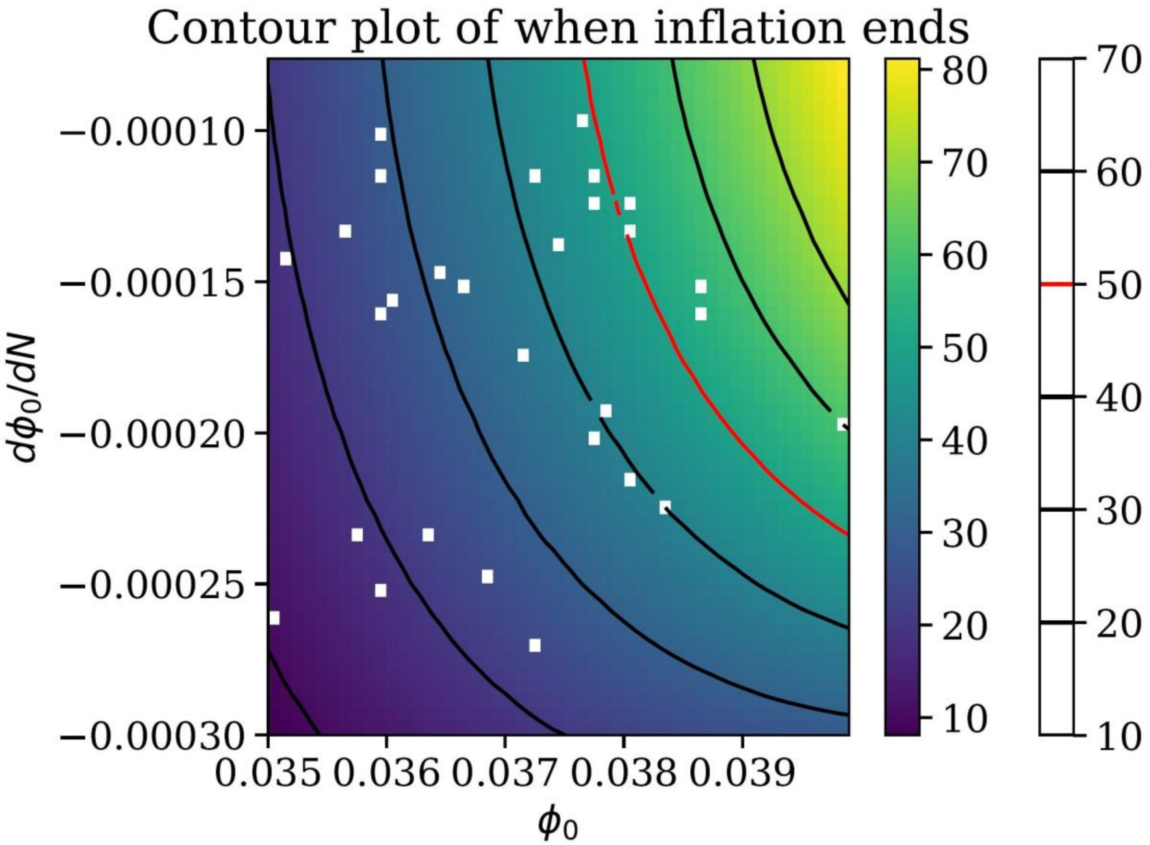




New Higgs Phase Space

- Some convergence issues in the right figure, left figure is converged!





Two Candidates, Two Potentials

Old Higgs Inflation

- Not Coupled:

$$\Phi = \Psi$$

- Assumes GR holds for all time
- Ruled out as we have previously

New Higgs Inflation

- Non-Minimally Coupled:

$$\Phi \neq \Psi$$

- Assumes GR may not hold prior to CMB
- Allows for a shear between the space-like Φ and time-like Ψ potentials

FRW Metric which describes spacetime in cosmology:

$$ds^2 = a(\tau)^2[-(1+2\Psi)d\tau^2 + (1-2\Phi)(dr^2 + r^2 d\Omega^2)]$$

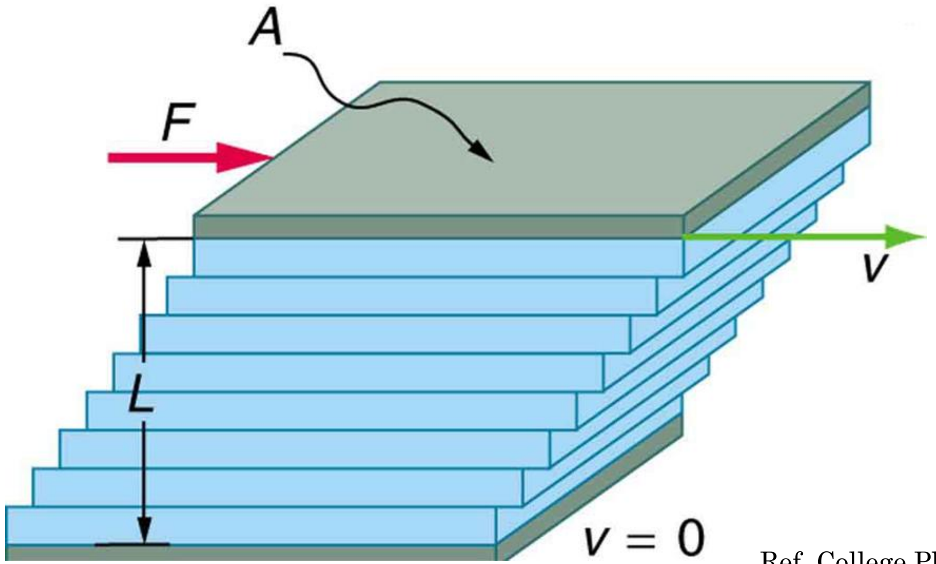
Early Universe Shear Stress

Fourier sum over matter species i:

$$k^2\Phi = -4\pi G a^2 \sum_i \rho_i \Delta_i$$

$$k^2(\Psi - \Phi) = -12\pi G a^2 \sum_i \rho_i (1 + w_i)\sigma_i$$

If shear $\sigma_i = 0$, then $\Psi = \Phi$.



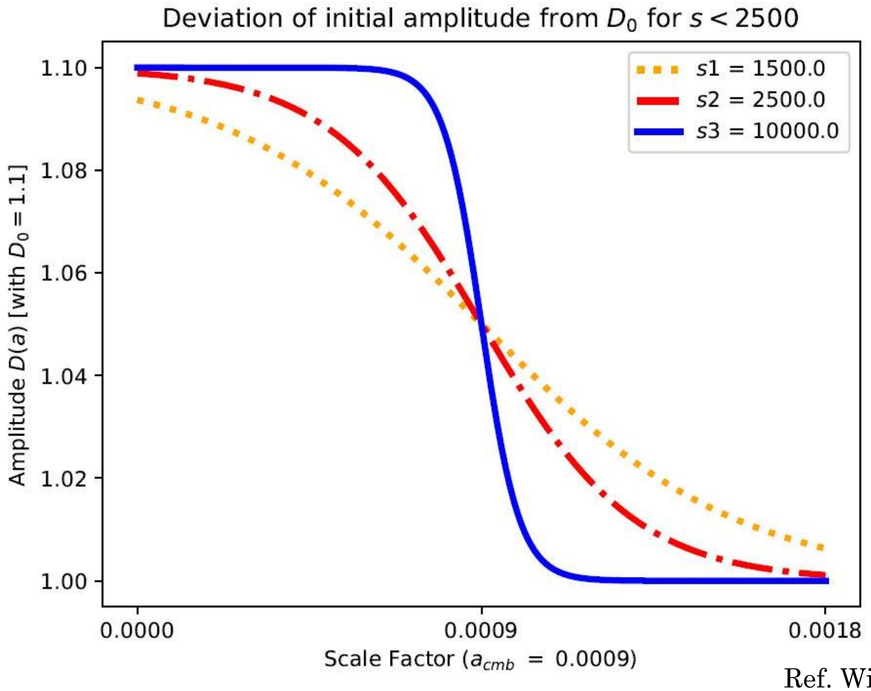
Ref. College Physics: OpenStax

Previous New Higgs Research

- Work by William Koster tested a simplified model such that:

$$\Phi = (2D + 1)\Psi$$

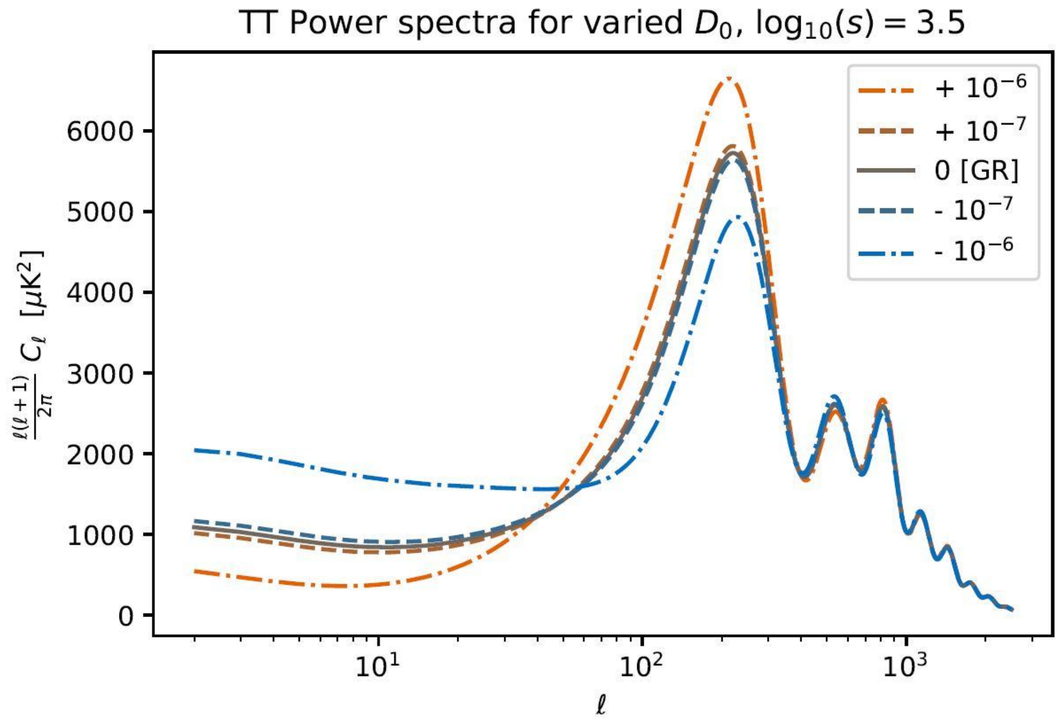
- Difference between the potentials drops off as a Tanh function

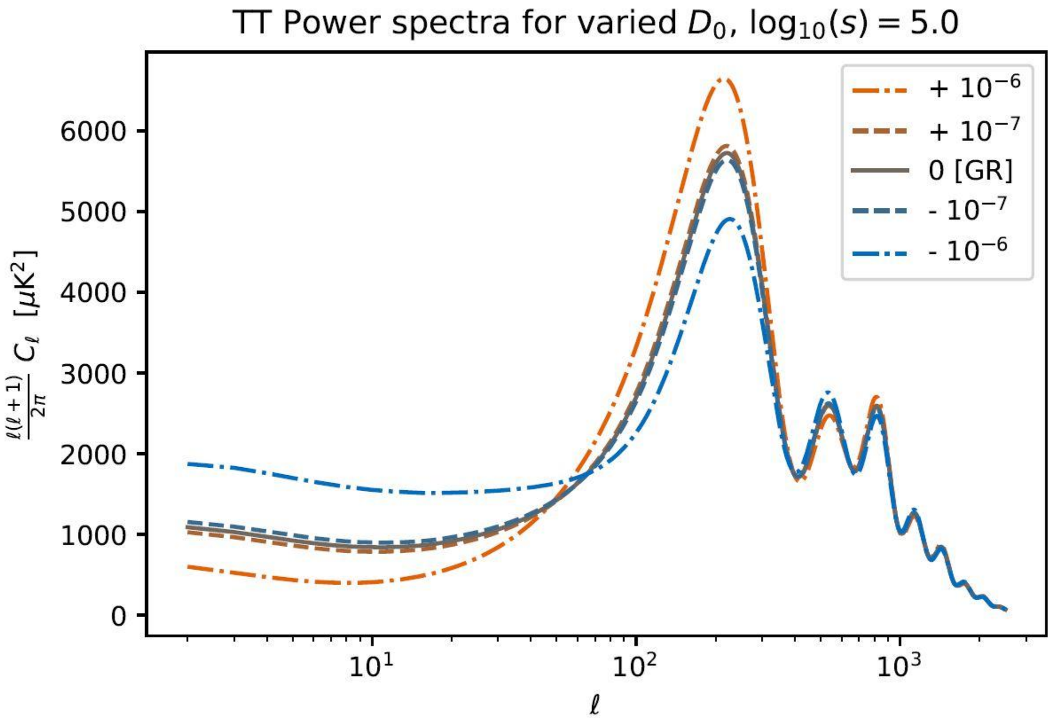


Ref. Winn Koster's Thesis

Previous New Higgs Results

- Affect on CMB for different s (rate of decay) values

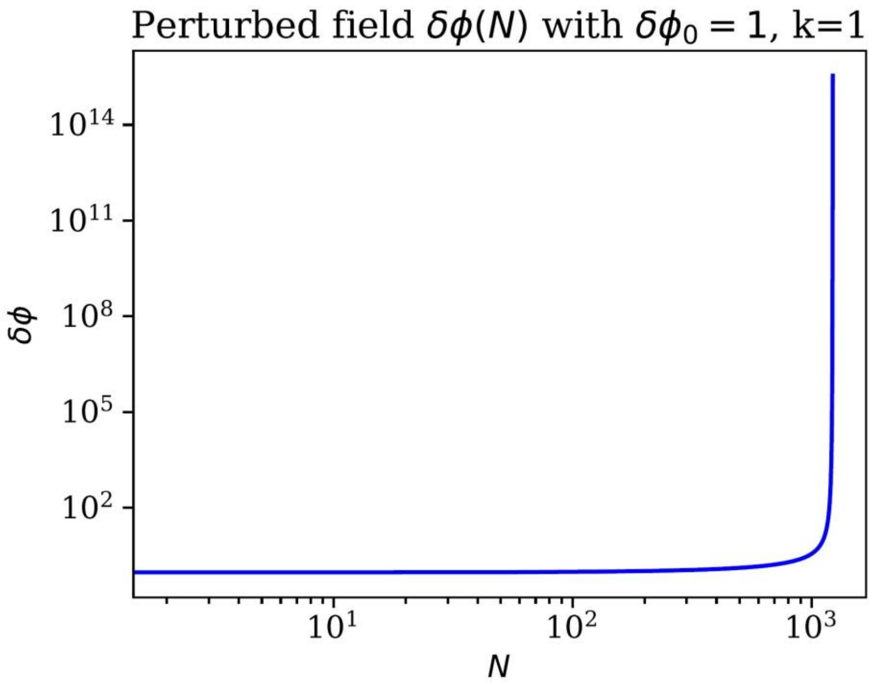


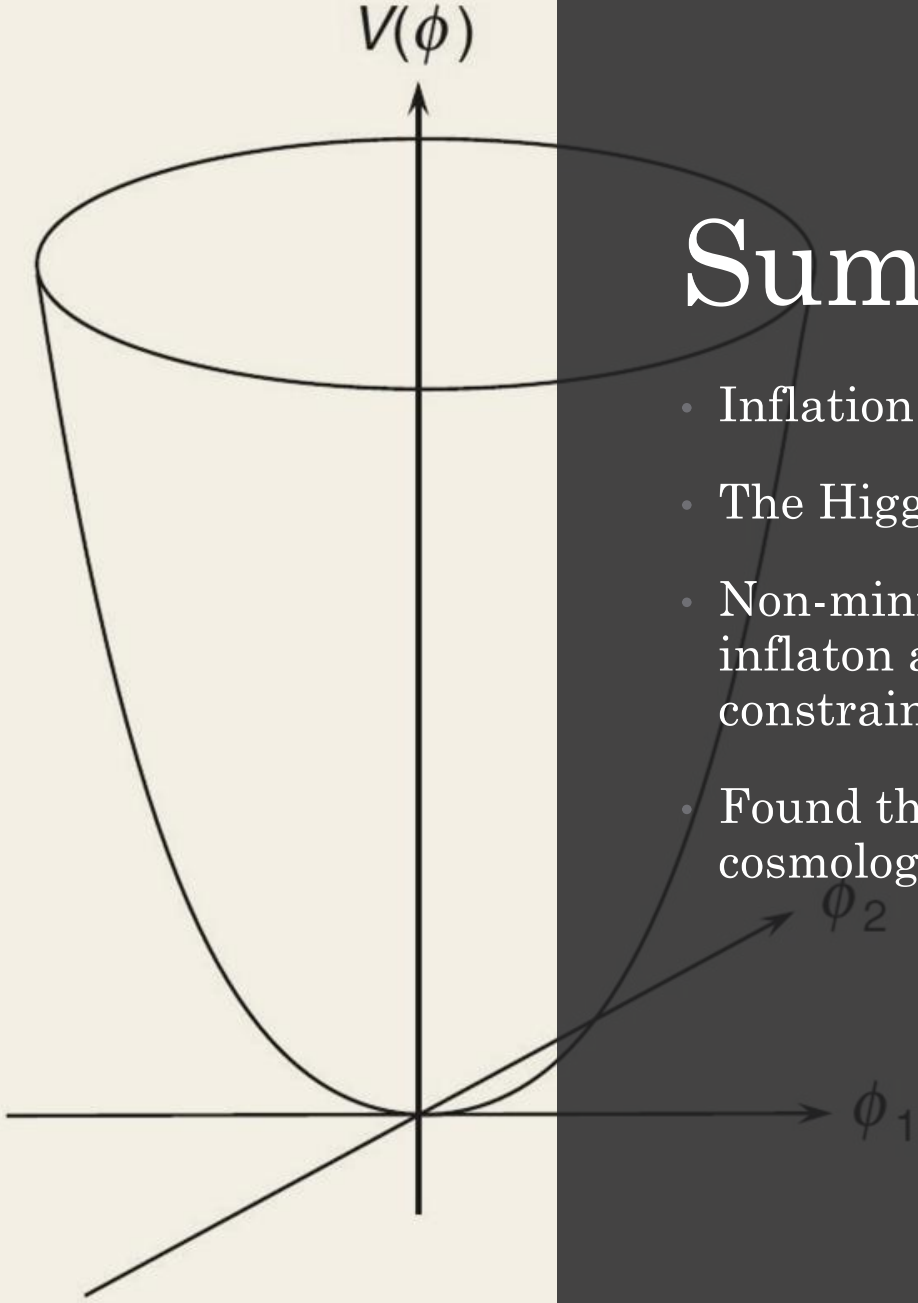


Ref. Winn Koster's Thesis

What Comes Next: Perturbations

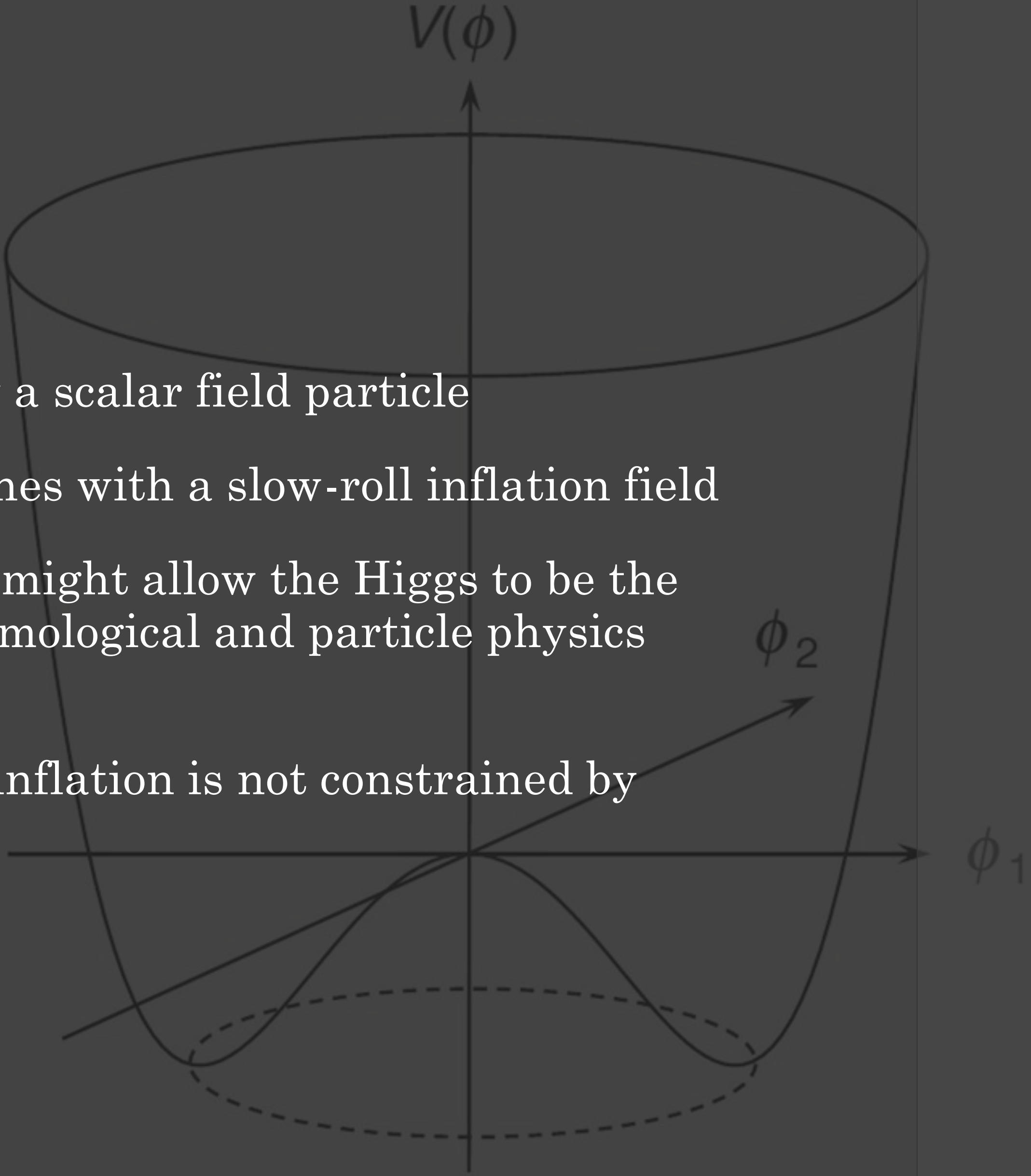
- End goal is to find relationship between Φ and time-like Ψ during the early universe
- To do so, model perturbations around homogeneous equations
- Need to rigorously solve initial conditions for Old and New Higgs models

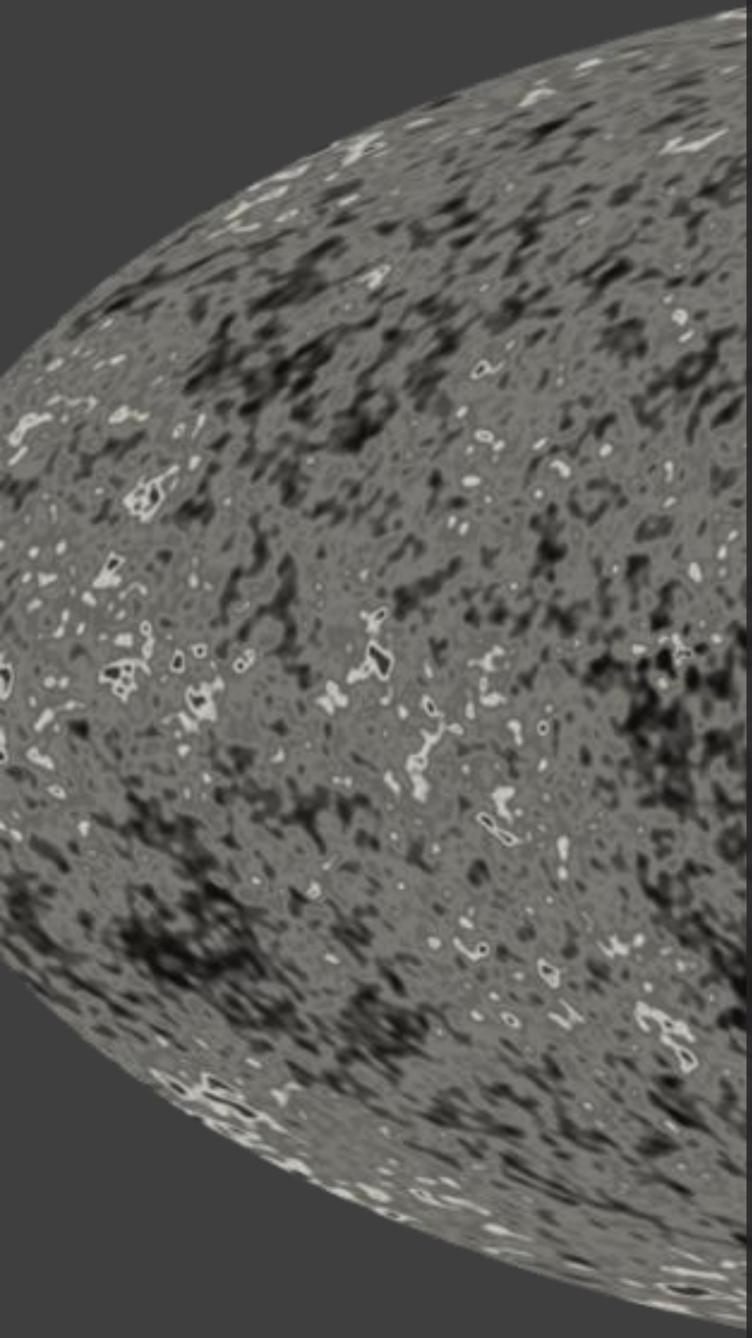




Summary

- Inflation is modeled by a scalar field particle
- The Higgs Boson matches with a slow-roll inflation field
- Non-minimal coupling might allow the Higgs to be the inflaton and match cosmological and particle physics constraints
- Found that Old Higgs inflation is not constrained by cosmology





Acknowledgements

- Thank you to my advisor Daniel Grin.
- David Kaiser for his Higgs Inflation notes
- Alan Guth for pioneering and developing this subject
- Our collaborators
- To Haverford KINSC office for funding me

- Questions?